

Math 45 SSM 2/e 4.3 Solving Systems Using Elimination (or Addition)

Objectives: 1) Solve linear systems using elimination method.

This is the third method for doing the same kind of problem

1) Graphing

2) Substitution

3) Elimination (also called Addition) Method

Solve the linear system by elimination and classify.

$$\textcircled{1} \quad \begin{cases} x+y=13 & A \\ 2x-y=-7 & B \end{cases}$$

Goal of Elimination Method

Add 2 equations together so that one variable adds to zero and disappears (is eliminated).

Step 1: Notice that $+y$ in first equation $-y$ in second equation add to zero.

Step 2: Add equations.

$$\begin{array}{r} x+y=13 \\ 2x-y=-7 \\ \hline 3x \quad = 6 \end{array}$$

Step 3: Solve the result for remaining variable

$$\frac{3x}{3} = \frac{6}{3}$$

$$x=2.$$

Step 4: substitute result into either equation.

$$\begin{array}{r} 2+y=13 \quad A \\ -2 \quad \quad \quad -2 \\ \hline \end{array}$$

$$y=11$$

Step 5: write solution and classify

$$(2, 11)$$

consistent independent

Note: Step 1 is not always so easy!

We may need to multiply one equation by a useful number -- or maybe both equations by useful numbers.

$$\textcircled{2} \begin{cases} 2x + 3y = -6 & \leftarrow A \\ -4x + 5y = -21 & \leftarrow B \end{cases}$$

CAUTION

If we add equations as given,
we get $-2x + 8y = -27$

No variable is eliminated 😞.

Do not do this.

step 1: Consider options

option 1: To eliminate x
we have $2x$ and $-4x$.

If we multiply A by 2 , we'll have $2(2x) = 4x$
and $4x + (-4x) = 0$.

option 2: To eliminate y .

We have $3y$ and $5y$.

If we multiply $3y$ by 5 we'll have $15y$

If we multiply $5y$ by (-3) we'll have $-15y$

We'd have to multiply both equations! 😞.

Choose option 1: (multiply only one equation)

$$\begin{array}{ll} 2x + 3y = -6 & \text{multiply All of eqn A by 2} \\ 2(2x) + 2(3y) = 2(-6) & \text{(using laws of algebra)} \end{array}$$

$$\begin{array}{ll} 4x + 6y = -12 & \leftarrow \text{new A} \\ -4x + 5y = -21 & \leftarrow \text{same B} \end{array}$$

step 2:

$$\frac{11y}{11} = \frac{-33}{11}$$

step 3:

$$y = -3$$

step 4:

$$\begin{array}{rcl} 2x + 3y & = & -6 \\ 2x + 3(-3) & = & -6 \\ 2x - 9 & = & -6 \\ \quad +9 & \quad +9 & \\ \hline 2x & = & 3 \\ \frac{2x}{2} & \quad \frac{3}{2} & \\ \hline x & = & \frac{3}{2} \end{array}$$

step 5

Solution $(\frac{3}{2}, -3)$
consistent independent

$$\textcircled{3} \begin{cases} 3y + 2x = 3 & A \\ 3x + 5y = 7 & B \end{cases}$$

step 0: Write in order:

$$\begin{cases} 2x + 3y = 3 & A \\ 3x + 5y = 7 & B \end{cases}$$

CAUTION:

Equations must be in standard form!

Step 1 consider options.To eliminate x :

multiply A by 3 $3(2x) = 6x$
and multiply B by -2 $-2(3x) = -6x$

To eliminate y :

multiply A by 5 $5(3y) = 15y$
and multiply B by -3 $-3(5y) = -15y$

choose option 1 (multiply by smaller numbers)

$$\begin{aligned} 3 \cdot 2x + 3 \cdot 3y &= 3 \cdot 3 & A \times 3. \\ 6x + 9y &= 9 & \text{new A.} \end{aligned}$$

$$\begin{aligned} -2 \cdot 3x + (-2) \cdot 5y &= (-2) \cdot 7 & B \times (-2) \\ -6x - 10y &= -14 & \text{new B.} \end{aligned}$$

$$\begin{aligned} 6x + 9y &= 9 \\ -6x - 10y &= -14 \end{aligned}$$

step 2: $-y = -5$

step 3: $y = 5$

step 4: $2x + 3y = 3$

$$2x + 3(5) = 3$$

$$2x + 15 = 3$$

$$\begin{array}{r} -15 \quad -15 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{-12}{2}$$

$$x = -6$$

step 5:solution $(-6, 5)$

consistent independent

Note: You can put the negative on either multiplier for either option

Math 45 4.3 cont p.4

$$\textcircled{4} \begin{cases} 4x - 6y = -5 \\ 6x - 9y = 2 \end{cases}$$

step 1: option 1 eliminate x

$$\begin{array}{l} \text{multiply A} \times 3 \quad 3(4x) = 12x \\ \text{and multiply B} \times (-2) \quad -2(6x) = -12x \end{array}$$

option 2 eliminate y

$$\begin{array}{l} \text{multiply A} \times 3 \quad 3(-6y) = -18y \\ \text{and multiply B} \times (-2) \quad (-2)(-9y) = 18y \end{array}$$

Both options are the same. (Not just equivalent, but the same.)

$$3(4x) - 3(6y) = 3(-5) \quad \text{A by 3.}$$

$$12x - 18y = -15 \quad \text{new A}$$

$$(-2)(6x) - (-2)9y = (-2)2 \quad \text{B} \times (-2)$$

$$-12x + 18y = -4$$

$$12x - 18y = -15$$

$$-12x + 18y = -4$$

$$\hline 0 + 0 = -19$$

* BOTH VARIABLES DISAPPEAR *

This is correct work.

$0 \neq -19$ is false

no solution
inconsistent

Note: goal is
the LCM of
the coefficients

Math 45 4.3 cont p.5

$$\textcircled{5} \begin{cases} 3x + 4y = -6 & A \\ \frac{3}{2}x + 2y = -3 & B \end{cases}$$

Step 0: clear fractions from B. Mult B x 2

$$2\left(\frac{3}{2}x\right) + 2(2y) = 2(-3)$$

$$3x + 4y = -6 \quad \text{new B.}$$

system

$$3x + 4y = -6 \quad A$$

$$3x + 4y = -6 \quad \text{new B.}$$

These are the same equation!

Multiply equation A by (-1):

$$-3x - 4y = 6$$

$$3x + 4y = -6$$

$$\hline 0 + 0 = 0$$

* VARIABLES DISAPPEAR *

This is correct work.

$0=0$ is true.

all points on line are solutions
consistent dependent

- ⑥ At the ballpark, one family buys 7 hot dogs and 5 drinks for 38.25.
Another buys 5 hot dogs and 4 drinks for 28.50.
Find the price of a hotdog and the price of a drink.

Let x = price of hot dog
 y = price of drink.

First family spent $7x + 5y = 38.25$
Second family spent $5x + 4y = 28.50$

Solve system $\begin{cases} 7x + 5y = 38.25 & A \\ 5x + 4y = 28.50 & B \end{cases}$

options: substitution $\Rightarrow$ fractions for all choices. (X)
eliminate x : mult A by (+5)
mult B by (-7)

eliminate y : mult A by 4
mult B by (-5)

choose eliminate y : (smaller numbers)

$$4(7x) + 4(5y) = 4(38.25)$$

$$28x + 20y = 153 \quad \text{new A}$$

$$-5(5x) + (-5)(4y) = (-5)(28.50)$$

$$-25x - 20y = -142.5 \quad \text{new B}$$

$$\begin{array}{r} 28x + 20y = 153 \\ -25x - 20y = -142.5 \\ \hline \end{array}$$

$$3x + 0 = 10.5$$

$$\frac{3x}{3} = \frac{10.5}{3}$$

$$x = 3.5$$

hot dogs cost \$3.50

substitute into B:

$$5(3.5) + 4y = 28.50$$

$$17.5 + 4y = 28.5$$

$$\begin{array}{r} 17.5 + 4y = 28.5 \\ -17.5 \quad -17.5 \\ \hline \end{array}$$

$$\frac{4y}{4} = \frac{11}{4}$$

$$y = 2.75$$

drinks cost \$2.75

Math 45 4.3 cont p.7

Solve by elimination and classify.

$$\textcircled{7} \begin{cases} \frac{x}{3} + \frac{y}{5} = 2 & A \\ \frac{x}{3} - \frac{2y}{5} = -1 & B \end{cases}$$

clear fractions - mult A by LCD=15

$$\frac{5}{15} \cdot \frac{x}{3} + \frac{3}{15} \cdot \frac{y}{5} = 15 \cdot 2$$

$$5x + 3y = 30 \quad \text{new A}$$

clear fractions - mult B by LCD=15

$$\frac{5}{15} \left(\frac{x}{3} \right) + \frac{3}{15} \left(-\frac{2y}{5} \right) = 15(-1)$$

$$5x - 6y = -15 \quad \text{new B.}$$

$$\begin{cases} 5x + 3y = 30 & \text{new A} \\ 5x - 6y = -15 & \text{new B} \end{cases}$$

mult A by -1 to eliminate x.

$$-5x - 3y = -30$$

$$5x - 6y = -15$$

$$\begin{array}{r} -5x - 3y = -30 \\ 5x - 6y = -15 \\ \hline -9y = -45 \\ -9 \quad -9 \end{array}$$

$$y = 5$$

substitute into new A

$$5x + 3(5) = 30$$

$$5x + 15 = 30$$

$$\underline{-15} \quad \underline{-15}$$

$$\frac{5x}{5} = \frac{15}{5}$$

$$x = 3$$

Solution (3, 5)
consistent independent

$$\textcircled{B} \begin{cases} 12y = 8x + 3 & A. \\ -2 + 4(3y - 2x) = 5 & B. \end{cases}$$

Step 0: Simplify and organize:

$$12y = 8x + 3$$

$$0 = 8x - 12y + 3$$

$$-3 = 8x - 12y$$

$$8x - 12y = -3 \quad A$$

$$-2 + 4(3y - 2x) = 5$$

$$-2 + 12y - 8x = 5$$

$$\begin{array}{r} -8x + 12y - 2 = 5 \\ \quad \quad \quad +2 \quad +2 \end{array}$$

$$-8x + 12y = 7 \quad B$$

$$\begin{cases} 8x - 12y = -3 & A \\ -8x + 12y = 7 & B \end{cases}$$

Step 1, 2, 3:

$$0 + 0 = 4$$

$$0 = 4$$

* VARIABLES DISAPPEAR *
false result

No solution
inconsistent

Math 45 4.3 cont p.9

$$\textcircled{9} \begin{cases} -2.4x - 0.4y = 0.32 & A \\ 4.2x + 0.6y = -0.54 & B \end{cases}$$

option 1: clear decimals first

mult A by 100.

$$-240x - 40y = 32$$

divide by 8

$$-30x - 5y = 4 \quad \text{new A}$$

mult B by 100

$$420x + 60y = -54$$

divide by 6

$$70x + 10y = -9 \quad \text{new B.}$$

$$\begin{cases} -30x - 5y = 4 & \text{new A} \\ 70x + 10y = -9 & \text{new B} \end{cases}$$

mult new A by 2 to eliminate y

$$\begin{array}{r} -60x - 10y = 8 \\ 70x + 10y = -9 \\ \hline \end{array}$$

$$\frac{10x}{10} = \frac{-1}{10}$$

$$x = -1 = -\frac{1}{10}$$

subst into new A:

$$-30(-1) - 5y = 4$$

$$30 - 5y = 4$$

$$-5y = 1$$

$$y = -\frac{1}{5} = -0.2$$

$(-1, -0.2)$
consistent
independent

or

$(-\frac{1}{10}, -\frac{1}{5})$
consistent
independent

option 2 use
decimals.

mult A $\times 3$
mult B $\times 2$ } to elim y.

$$\begin{array}{r} -7.2x - 1.2y = 0.96 \\ 8.4x + 1.2y = -1.08 \\ \hline \end{array}$$

$$\frac{1.2x}{1.2} = \frac{-1.2}{1.2}$$

$$x = -1$$

subst

$$-2.4(-1) - 0.4y = 0.32$$

$$2.4 - 0.4y = 0.32$$

$$-0.4y = 0.08$$

$$y = -0.2$$

Solution

$(-1, -0.2)$
consistent
independent